Remarks on the Classical Size of D-Branes

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We discuss different criteria for "classical size" of extremal Dirichlet p-branes in type-II supergravity. Using strong–weak coupling duality, we find that the size of the strong-coupling region at the core of the $(p < 3)$–branes, is always given by the asymptotic string-scale, if measured in the weakly coupled dual string-metric. We also point out how the eleven-dimensional Planck scale arises in the classical 0-brane solution, as well as the ten-dimensional Planck scale in the D-instanton solution.

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One of the interesting properties of Dirichlet-branes (D-branes) [1], [2], is their sharp character in space-time. Since they are perturbatively defined as a boundary condition for open-string propagation, they have no "bare" size. Still, they interact at a perturbative level via virtual-string exchange, and there is an effective string-scale "halo" which represents a minimun size for perturbative string probes (with the notable exception of the D-instanton [3]). A minimun distance of the order of the string scale is also singled out when considering the short-distance interactions of branes and anti-branes [4], [5], [6], [7], due to a local analogue of the well-known Hagedorn phenomenon (see for example [8]).

On the other hand, if we use D-branes themselves as probes [9], [10], [11], [12], we can isolate sub-stringy scales in the slow scattering of D-branes. Such sub-string scales were predicted on general grounds in [13].

In this note we collect some heuristic arguments based on the analysis of low-energy supergravity solutions. We will consider the relevant dynamical scales as being associated with the strong-coupling dynamics measured by the local string coupling $\lambda(r) = e^{\phi(r)}$, where $\phi(r)$ is the dilaton expectation value in the configuration of interest. Using string dualities locally, we can always turn a region of extreme strong coupling into a weakly coupled region by an appropriate change of variables. Therefore, we will be concerned with the points of "trully strong coupling", namely those with a local string coupling of order unity $\lambda(r) \sim \mathcal{O}(1)$. In this way, we include non-perturbative effects through the non-linearities of the low-energy supergravity theory.

At first sight, these considerations have at best a heuristic value, because the low-energy supergravity solution should not be trusted at short distances. However, we should not forget that the supergravity solutions are protected by supersymmetry. Moreover, because of the strong-weak coupling dualities of the ten-dimensional supergravities, we never have to consider a local region of space-time at extreme strong coupling. For these reasons, we think that the classical analysis, if properly interpreted, is capable of capturing the essential scales, at least in order of magnitude.

The relevant bosonic terms in the string-frame supergravity action are (excluding the $p = 3$ case):

$$S = \int \sqrt{|g|} e^{-2\phi} \left(R - 4(\partial \phi)^2 \right) + \int \sqrt{|g|} \frac{e^{2a\phi}}{2(8-p)!} F_{8-p}^2. \tag{1}$$

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Here the parameter $a = 0$ corresponds to R–R backgrounds in type-II, and $a = -1$ corresponds to the heterotic low-energy theory. The R–R solutions, in magnetic form, with a vanishing asymptotic dilaton, are given by [14], [15]:

$$ds^2 = f^{-1/2}dx_L^2 + f^{1/2}\left(dr^2 + r^2 d\Omega_{8-p}^2\right)$$
$$e^{-2\phi'} = f^{\frac{p-3}{2}}$$
$$F'_{8-p} = C_p\, d\Omega_{8-p} \tag{2}$$
$$f = 1 + \frac{1}{7-p}\frac{C_p}{r^{7-p}},$$

with dx_L the coordinate elements in the world-volume directions, and r the transverse radial coordinate. The constant C_p is proportional to the charge Q, or total number of coincident branes. We can switch on an asymptotic dilaton and coupling constant $\lambda_\infty = e^{\phi_\infty}$ by the substitutions $\phi = \phi' + \phi_\infty$ and $F' = \lambda_\infty^{a+1}F$. The action becomes

$$S = \frac{1}{\lambda_\infty^2}S(\text{primed fields}) \sim \frac{C_p}{\lambda_\infty^2}.$$

Therefore, for R–R branes we need to take $C_p \sim \lambda_\infty Q$, in order to have the correct R–R scaling of the action $S \sim Q/\lambda_\infty$.

The same result follows from the ADM definition of tension (see for example ref. [16]). Consider a completely wrapped R–R p-brane, which looks like a black hole in $d = 10 - p$ dimensions. The corresponding string-frame mass can be calculated using T and U-duality:

$$M_p = \frac{QR^p}{\lambda_\infty\,(\alpha')^{\frac{p+1}{2}}}, \tag{3}$$

for a symmetric torus of radius R. This is the same mass that one finds in the so-called "modified Einstein frame", which decouples the dilaton, but coincides asymptotically with the string frame

$$(g_{\mu\nu})_{\text{str}} = e^{\frac{4\phi'}{d-2}}(g_{\mu\nu})_{ME}. \tag{4}$$

Using the solutions above, we have the following ADM matching at large distances:

$$-(g_{00})_{ME} = f^{\frac{3-d}{d-2}} \rightarrow 1 - \frac{1}{d-2}\frac{C_p}{r^{d-3}} = 1 - \frac{1}{(d-2)\,|S^{d-2}|}\frac{16\pi G_d M_p}{r^{d-3}}, \tag{5}$$

with $|S^{d-2}|$ the volume of the unit $(d-2)$-sphere and G_d the d-dimensional Newton constant, given by the standard Kaluza–Klein reduction, $G_d = G_{10}/(2\pi R)^p$, from the ten-dimensional Newton constant. The latter is fixed by string-fivebrane Dirac duality to be $G_{10} = 8\pi^6\alpha'^4\lambda_\infty^2$. With this information, it is easy to solve for C_p with the result

$$C_p = \lambda_\infty Q\,\frac{(2\pi\sqrt{\alpha'})^{7-p}}{|S^{8-p}|}. \tag{6}$$

The upshot of the preceding discussion is simply that we must use the classical metric in (2) with $C_p \sim Q\lambda_\infty$ and a local string coupling

$$\lambda(r) = \lambda_\infty \, f(r)^{\frac{3-p}{4}}. \tag{7}$$

For $p < 3$, the string metric at the core exhibits a neck followed by an infinite non-compact space extending an infinite proper distance, i.e. $\int dr \sqrt{g_{rr}}$ diverges as we approach the origin. The volume of the transverse sphere S^{8-p} at the point r is given by

$$V_\Omega(r) = \left(\sqrt{r^2 \, f^{1/2}(r)} \right)^{8-p} \cdot |S^{8-p}|,$$

and scales as r^{8-p} for large r, and as $r^{(p-5)(8-p)}$ close to the origin. Therefore, there is a minimun transverse volume of the neck, scaling as $V_\Omega^{\min} \sim (Q\lambda_\infty)^{\frac{8-p}{7-p}}$, thereby defining an effective proper radius[1]

$$R_{\min} \sim (Q\lambda_\infty)^{\frac{1}{7-p}}. \tag{8}$$

At this point $f_{\min} \sim \lambda_\infty^0 \sim \mathcal{O}(1)$, and the local string coupling is of the same order as the asymptotic coupling. This means that, in some sense, we can assign a small size to the D-brane soliton, within the realm of perturbative string theory. The more interesting scales, however, are associated to the region of local strong coupling.

For the self-dual 3-brane, the dilaton is uniform and asymptotic weak coupling ensures weak coupling throughout space. For $p > 3$, the situation is even better, since the horizon region is always weakly coupled. On the other hand, for $p < 3$ we have strong coupling at the origin. Thus, the other natural scale we can define is determined by the point r_c at which the coupling becomes strong; $\lambda(r_c) \sim \mathcal{O}(1)$. It scales as

$$r_c \sim \lambda_\infty^{\frac{1}{3-p}} \, Q^{\frac{1}{7-p}}. \tag{9}$$

An analogous scale is defined by requiring the effective open-string coupling $\lambda_o \sim \lambda(r)Q$ to be of order unity:

$$r_o \sim (Q\lambda_\infty)^{\frac{1}{3-p}}. \tag{10}$$

In view of the results of ref. [12], it is very suggestive that (9) and (10), defined as the radial coordinate where strong-coupling effects take place, gives the eleven-dimensional

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[1] Unless otherwise specified, lengths are measured in units of the string length $\sqrt{\alpha'}$.

Planck length for the case of the 0-branes; $\ell_{11} \sim \lambda_\infty^{1/3} \sqrt{\alpha'}$. Other interesting cases are the D-instanton $(p = -1)$, for which the strong-coupling radial coordinate is of the order of the ten-dimensional Planck scale[2], $\ell_{10} \sim \lambda_\infty^{1/4} \sqrt{\alpha'}$, and the D-string $(p = 1)$, whose "size" under this criterium scales as $\lambda_\infty^{1/2} \sqrt{\alpha'}$, again in agreement with the dynamical analysis of ref. [12].

We regard these agreements as significant, even if (9) and (10) do not define a proper distance in any of the non-linearly corrected string metrics for $p < 3$. At the end of this note we will comment on the significance of the scales defined by r_c and r_o. However, at this stage, it is more natural to perform a strong-weak coupling duality transformation to rewrite the geometry of the core region in terms of weakly coupled variables. Since such duality transformations involve a Weyl rescaling of the string metric, the infinite non-compact space at the core is replaced by a finite patch with a pointlike singularity, which can be matched to a delta-function source. It is then natural to define a strong-coupling radius as the proper distance from the origin to the self-dual point where the local string coupling is of order unity, i.e. the point of "trully strong coupling", measured in the weakly coupled string metric.

In the case of $p = -1, 1$ branes, we have a type II-B theory, and the dual metric is found by the usual requirement that the Einstein metric is invariant, where, in ten dimensions $e^{-\phi/2} g_{\mu\nu} = g^E_{\mu\nu} = e^{-\tilde{\phi}/2} \tilde{g}_{\mu\nu}$, and $\phi = -\tilde{\phi}$. There is an interesting cancellation of all string coupling powers, which results in a proper distance proportional to the D-brane charge;

$$
\tilde{L} = \int_0^{r_c} dr \sqrt{\tilde{g}_{rr}} = \int_0^{r_c} dr \lambda(r)^{-\frac{1}{2}} \sqrt{g_{rr}} \sim \lambda_\infty^{-\frac{1}{2}} \int_0^{r_c} dr f(r)^{\frac{p-1}{8}} \sim
$$
$$
\sim \lambda_\infty^{-\frac{1}{2} + \frac{p-1}{8}} Q^{\frac{p-1}{8}} r_c^{\frac{(7-p)(1-p)}{8} + 1} \sim Q \lambda_\infty^0 \sim Q. \tag{11}
$$

On the other hand, when considering $p = 0, 2$ branes, we have a type II-A theory, and the strong-coupling limit is governed by eleven-dimensional supergravity. The mapping between the II-A string metric and the ten-dimensional part of the Sugra_{11} metric is [18]

$$
\tilde{g}_{rr} = e^{-\frac{2\phi}{3}} g_{rr},
$$

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[2] This agrees with the dynamical analysis of [17].

with ϕ the II-A dilaton. Remarkably, the powers of λ_∞ still conspire to cancel the coupling dependence:

$$\tilde{L} = \int_0^{r_c} dr \sqrt{\tilde{g}_{rr}} = \int_0^{r_c} dr \lambda(r)^{-\frac{1}{3}} \sqrt{g_{rr}} = \lambda_\infty^{-\frac{1}{3}} \int_0^{r_c} dr f(r)^{\frac{p}{12}} \sim$$
$$\sim \lambda_\infty^{-\frac{1}{3}+\frac{p}{12}} \, Q^{\frac{p}{12}} \, r_c^{\frac{p(p-7)}{12}+1} \sim Q\lambda_\infty^0 \sim Q. \tag{12}$$

Thus, the so-defined strong-coupling radius of R–R p-branes is always $\tilde{L}_{RR} \sim \sqrt{\alpha'} = \ell_{\text{str}}$, where ℓ_{str} is the string scale set by the asymptotic weakly coupled string theory. We find it remarkable that this behaviour also extends to the type-IIA case, where the dual, weakly coupled metric, is *not* a string sigma-model metric.

This situation also applies to the case of the heterotic 5-brane, whose metric is

$$ds^2 = dx_L^2 + \left(1 + \frac{r_+^2}{r^2}\right)\left(dr^2 + r^2 d\Omega_3^2\right),$$

and the local string coupling, $\lambda(r) = \lambda_\infty \sqrt{1 + r_+^2/r^2}$. In this case we have to choose $r_+ \sim \lambda_\infty^0$, because the action must have an NS scaling $S \sim \lambda_\infty^{-2}$. Then, the strong-coupling $\lambda(r_c) \sim \mathcal{O}(1)$ point lies at $r_c \sim \lambda_\infty$, and the proper distance in the dual type-I metric [18]

$$\tilde{L}_{NS} = \int_0^{r_c} dr \lambda(r)^{-\frac{1}{2}} \sqrt{g_{rr}} \sim \lambda_\infty^{-1} \int_0^{r_c} dr \sqrt{\lambda(r)} \sim \sqrt{\frac{r_c}{\lambda_\infty}} \sim \lambda_\infty^0 \sim \mathcal{O}(1).$$

It is interesting to notice that the same results follow in the Einstein frame:

$$L_E = \int_0^{r_c} dr \sqrt{g_{rr}^E} = \int_0^{r_c} dr \lambda(r)^{-\frac{1}{4}} \sqrt{g_{rr}}.$$

One obtains $L_E \sim \sqrt{\alpha'}$ in both cases. Of course, this is just the Planck length in Einstein units. We can pass to the modified Einstein frame, which rescales the Einstein-frame lengths by the constant factor $\lambda_\infty^{1/4}$, and find the standard form of the Planck length in string units. Being equivalent to the string frame at large distances, it is not completely trivial that the modified Einstein frame should define a Planckian strong-coupling radius.

The short-distance scales defined by (9) and (10) correspond to proper distances in the flat metric that approaches the string metric at radial infinity. This is interesting because, in the D-brane picture, we have a flat Minkowski space-time all the way down to the D-brane core, where the Dirichlet boundary condition is enforced. In other words, the D-brane is really a source for closed string massless fields, and the back-reaction is not

taken into account in the definition of boundary states by "bare" Dirichlet conditions on the world-sheet fields.

Dynamically, the flat metric is appropriate for describing the slow scattering of test D-branes (see [12], [11]). To some extent, this is true even after we take into account the non-linearities. The action for a test D-brane in the non-linear background of another D-brane can be approximated by simply substituting the fields of (2) into the Born-Infeld action [19]. For a rigid, straight, D-brane in a physical gauge, i.e. $x^0 = t$, $\vec{x}_L = \vec{\sigma}$, $\vec{x}_T = \vec{x}_T(\tau)$, the factors of the profile function, $f(r)$, cancel out such that one finds

$$
\begin{aligned}
S_{BI} &= -\, C \int d\tau d^p\vec{\sigma}\; e^{-\phi} \sqrt{-\det(g_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu)} \\
&= -\frac{CV_p}{\lambda_\infty}\int d\tau\, \frac{1}{f} + \frac{CV_p}{\lambda_\infty}\int d\tau\, \frac{1}{2}\delta_{ij}\dot{x}_T^i \dot{x}_T^j + \mathcal{O}(\dot{\vec{x}}_T^4).
\end{aligned}
\tag{13}
$$

The first term is the action of the D-brane at rest, whereas the second term is simply the non-relativistic action for a slow-moving object with a mass $M = CV_p/\lambda_\infty$, in a space with flat transverse metric.

From the point of view of the closed-string sector, the D-brane boundary condition does not define an exact background, due to the non-vanishing tadpoles,

$$
\langle V_{\text{massless}}|B\rangle \neq 0,
\tag{14}
$$

of the massless closed-string states in the presence of the D-brane boundary state. This means that the series of string diagrams, with Dirichlet boundary conditions and free strings propagating in Minkowski space-time, is really an expansion around an approximate solution of the closed-string equations. To see how this explains some features of the Dirichlet-diagram expansion, consider for definiteness the case of a single D-instanton. The "tree" effective action is given by [20], [21], [22]:

$$
\Gamma_{\text{tree}} = \sum_{N=1}^{\infty} \frac{1}{N!} W_N,
\tag{15}
$$

where W_N is a sphere amplitude with N Dirichlet boundaries in the flat conformal field theory. If we perform a string field theory (SFT) decomposition as in [23], we can write (see also [12]):

$$
\Gamma_{\text{tree}} = \sum_N \sum_{\{\alpha\}} \frac{1}{N!} W_{\alpha_1 \cdots \alpha_N} \prod_{i=1}^{N} \frac{1}{-\partial^2 + M_{\alpha_i}^2} \langle V_{\alpha_i}^* |B\rangle,
\tag{16}
$$

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where $W_{\{\alpha\}}$ denotes the generalized SFT irreducible vertex with the indices α_i labeling states defined in the flat conformal field theory[3], and $\langle V_{\alpha_i}^* | B \rangle$ is an off-shell SFT generalization of the perturbative tadpoles in (14). Using now the fact that correlators of vertex operators give directly the amputated string field theory amplitudes, we can write (16) in the form

$$\Gamma_{\text{tree}} = \sum_{\{\alpha\}} \frac{1}{N!} W_{\alpha_1 \cdots \alpha_N} \delta \Psi_{\alpha_1}^{c\ell} \cdots \delta \Psi_{\alpha_N}^{c\ell}. \tag{17}$$

Here

$$\delta \Psi_\alpha^{c\ell} \equiv \Psi_\alpha^{c\ell}(\text{inst}) - \Psi_\alpha^{c\ell}(\text{vac})$$

is the difference between the corresponding string field at the flat Minkowski CFT, and the exact closed-string instanton background. Therefore, if we interpret the flat, off-shell SFT vertices as functional derivatives,

$$W_{\alpha_1 \cdots \alpha_N} \sim \frac{\delta^{(N)} \Gamma_{\text{tree}}}{\delta \Psi_{\alpha_1} \cdots \delta \Psi_{\alpha_N}},$$

we find the expansion of the effective action around an approximate solution given by the flat CFT, with sharp Dirichlet boundary conditions at the D-brane core. In particular, this explains why we can calculate the instanton classical action by means of a diagrammatic computation (the disk diagram being the leading term), a fact that has no analogue in field theory, unless we construct the diagrammatic expansion around an approximate solution. Clearly, if the classical action of the approximate solution vanishes, the entire classical action of the exact instanton solution arises from tadpole contributions. Scattering amplitudes can be generated in the standard way, taking derivatives of the effective action with respect to linearized deformations of the flat CFT.

We think that the above string field theory interpretation of the Dirichlet diagrammatic expansion could shed some light on the dicotomy between the conformal field theory descriptions of NS–NS solitons, as opposed to the D-brane construction for R–R solitons. Indeed, the previous construction treats the R–R instantons in a more familiar way, from the field theoretical point of view.

Comming back to the issue of strong-coupling length scales, we conclude that the appropriate notion of "classical size" of D-branes depends on the probe we use to measure

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[3] Notice that we are inserting the full tower of massive closed-string states, and not only the massless ones, as would be appropriate at large distances.

geometry. If we consider closed string propagation in the background of a D-brane, non-linear effects induce a strong-coupling scale that, when measured in the weakly coupled metric, still corresponds to the string length $\sqrt{\alpha'}$, the size of the perturbative "halo". On the other hand, when considering the slow scattering of D-branes as a probe of geometry, the effective transverse metric is the flat metric (13). Thus, we can interpret the sub-stringy scales of [12] as the strong-coupling scales (9) and (10), determined directly from the classical dilaton profile in eq. (7).

1. Acknowledgements

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